Math Methods in Transistor Modeling: Condition Numbers for Parameter Extraction

Firman D. King, Peter Winson, Arthur D. Snider, Lawrence Dunleavy, and Deborah P. Levinson

***Abstract*—Condition numbers expressing the sensitivity of computed circuit element values to inaccuracies in S-parameter measurements are derived and evaluated for a standard small-signal MESFET model. The condition numbers shed light on the common difficulty experienced by transistor modelers in extracting accurate values for the input resistance. Other elements are also classified according to their sensitivity.**

***Index Terms*—Circuit modeling, MESFET's, parameter estimation.**

I. Mathematical Background

In the theory of computation *condition numbers* are dimensionless numbers expressing upper bounds for the relative error in the solution of a set of equations in terms of the relative accuracy of the data. A classical result is that errors in the solution to the matrix equation $\mathbf{Ax} = \mathbf{b}$ are induced by errors in the data in accordance with

$$\|\delta\mathbf{x}\|/\|\mathbf{x}\| \leq \|\mathbf{A}\|\|\mathbf{A}^{-1}\|\|\delta\mathbf{b}\|/\|\mathbf{b}\|$$

(matrix norms: $\|\mathbf{A}\| = \max\|Ax\|/\|x\|$) [1]. Thus, the condition number for the problem $\mathbf{Ax} = \mathbf{b}$ is $\|\mathbf{A}\|\|\mathbf{A}^{-1}\|$. Recent extensions of the theory have resulted in partitioned condition numbers and componentwise condition numbers, which help to distinguish the poorly determined components of the solution from the others [2], [3].

For a nonlinear set of equations $\mathbf{x} = \mathbf{f}(\mathbf{y})$, the influence of a relative error $\delta y_j/y_j$ in a data component on a solution component x_i would be measured by the ratio

$$|\delta x_i/x_i|/|\delta y_j/y_j| \approx |\partial x_i/\partial y_j| \times |y_j/x_i| \equiv \chi_{ij}. \tag{1}$$

A 1% error in the component y_j would produce approximately a χ_{ij}% error in x_i.

The accuracy of this approximation is limited by two considerations. First, it represents a linearization, which, of course, will have a limited range of validity. Additionally, the condition number or sensitivity χ_{ij} as calculated will depend on the data $\mathbf{y}$ from which $\mathbf{x}$ is calculated and at which the partial derivatives $|\partial x_i/\partial y_j|$ are evaluated. Thus, even the sensitivity has a sensitivity, and may be inaccurate due to errors in the data. Nevertheless, a large sensitivity calculated for a derived value x_i on a measurement y_j is usually taken as an indication that an accurate value x_i can be determined only with a very precise measurement of y_j, since either the sensitivity is indeed high or the errors, bad enough to render the sensitivity calculation unreliable, are likely to have resulted in an erroneous x_i.

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F. D. King, P. Winson, A. D. Snider, and L. Dunleavy are with the Department of Electrical Engineering, University of South Florida, Tampa, FL 33620-5350 USA.

D. P. Levinson is with the Department of Mathematics, Colorado College, Colorado Springs, CO 80903 USA.

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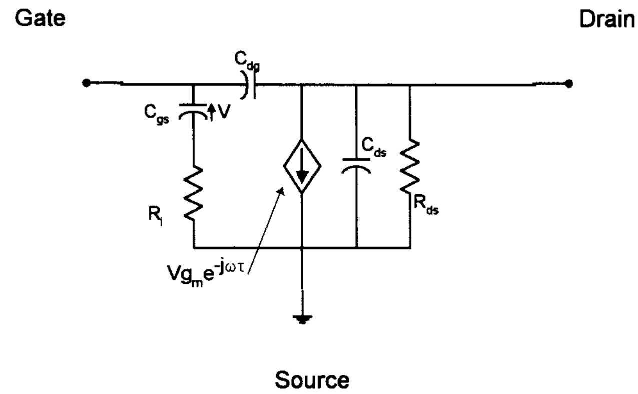

Fig. 1. Small-signal model of the MESFET.

II. Parameter Extraction for MESFET Models

Fig. 1 shows a standard small-signal model of a MESFET. The values of the circuit elements therein have traditionally been evaluated by measuring the scattering parameters ("S-parameters") and curve fitting. More recently, Berroth and Bosch have shown that the circuit elements can be computed explicitly from the admittance parameters ("Y-parameters"), whose (explicit) expressions in terms of the S-parameters are well known [4], [5]. Thus, we can derive formulas for the condition numbers of each of the circuit elements. The labor of this calculation is considerably reduced by utilization of MAPLE software.

The numbers which are directly measured in an S-parameter determination are the amplitudes $|S_{ij}|$ and phases $\angle S_{ij}$. The measuring device—the vector analyzer—may have the facility of reporting S values in either Cartesian (real/imaginary) or polar form, but only the latter are measured directly. Thus, the partials in the condition-number formulas (1) should be taken with respect to the polar variables. For example, for the input resistance, we have

$$\chi_{R_i,\,|S_{12}|} = \left|\frac{\partial R_i}{\partial |S_{12}|}\right| \times \left|\frac{|S_{12}|}{R_i}\right| \tag{2}$$

and

$$\chi_{R_i,\,\angle S_{12}} = \left|\frac{\partial R_i}{\partial \angle S_{12}}\right| \times \left|\frac{\angle S_{12}}{R_i}\right|. \tag{3}$$

Although the relationships between the polar-form S-parameters and the Cartesian-form S-parameters, between the Cartesian-form S-parameters and the Y-parameters, and between the Y-parameters and the circuit model parameters are all complicated and nonlinear, by using MAPLE software's symbolic mathematics capability, we were able to explicitly calculate symbolic expressions for all 64 partial derivatives and all 64 sensitivities in terms of the measurement data. Actual data from typical measurements were then inserted into these expressions and the sensitivities were evaluated.

Since explicit symbolic expressions were used, no recourse to the chain rule or to any numerical method was necessary. These expressions proved to be extremely complicated, covering many pages of computer printout. The effort to determine or utilize them would have been prohibitive without the symbolic mathematics software.

TABLE I
CIRCUIT PARAMETER SENSITIVITIES FOR TYPICAL DATA

	$\|S_{11}\|$	$\angle S_{11}$	$\|S_{12}\|$	$\angle S_{12}$	$\|S_{21}\|$	$\angle S_{21}$	$\|S_{22}\|$	$\angle S_{22}$
C_{gd}	-0.49	0.03	0.99	0.08	-0.01	0.08	-0.38	0.01
C_{ds}	1.14	-0.13	-2.38	-0.63	-1.77	-1.30	2.36	3.35
C_{gs}	-0.09	1.36	-0.36	-0.02	-0.26	-0.04	0.14	-0.003
R_i	-97.80	-0.96	-0.06	7.77	-0.27	16.55	0.21	0.31
g_m	-0.69	0.03	-0.01	0.05	0.99	0.13	-0.38	0.01
τ	679.20	-26.08	6.78	-53.28	8.70	-565.07	-9.25	-10.39
R_{ds}	-0.03	-0.02	0.04	-0.22	0.04	-0.46	2.16	-0.03
ξ	-97.90	0.40	-0.43	7.75	-0.53	16.50	0.35	0.30

III. RESULTS AND DISCUSSION

The sensitivities corresponding to a typical set of data are shown in Table I. For the devices tested at the WAMI Laboratory, University of South Florida, Tampa, for instance, workers have long recognized the input resistance R_i as a notoriously unstable parameter when determined numerically from S-parameter measurements. Our analysis pinpoints the difficulty. Table I shows a 97.80 sensitivity of R_i with respect to $|S_{11}|$ (for our device). Subject to the limitations and approximations described above, this would predict that a 1% error in the parameter $|S_{11}|$ causes an error of 98% in the element R_i. Since our equipment (a Hewlett-Packard HP8510B Network Analyzer) typically exhibits errors in the range of 3%–5% in this parameter under the test conditions,[1] R_i clearly cannot be determined accurately from S-parameter data.

The least reliably determined circuit element is the time-delay τ, for which our calculations show enormous sensitivities to both $|S_{11}|$ and $\angle S_{21}$. The parameter $|S_{11}|$, as stated above, is subject to 3%–5% measurement errors. The angle $\angle S_{21}$ for this data is approximately 167°, and phase measurement errors typically run around 5°,[1] or around 3%. (Indeed, phase errors are usually absolute, not relative quantities; one may wish to omit the normalizations in the phase factors for the phase condition numbers (3).) Thus, any determination of τ is completely swamped by its uncertainty. The difficulty in establishing accurate values for the time delay from S-parameter measurements is well known, and frequently τ is simply omitted from the model.

Condition-number analysis highlights the numerically unstable elements in the model, with respect to a specific experimental determination (i.e., via S-parameters). It identifies which particular S-parameters limit the accuracy of the results, and it tells us how accurately these must be measured in order to derive reliable circuit element values. These sensitivities are due to ill conditioning of the mathematical equations relating the circuit elements to the S-parameters. They are inherent in the model-plus-extraction procedure itself and are, therefore, algorithm independent.

However, they do not imply that the model is unverifiable. Alternative experimental procedures could conceivably finesse these high sensitivities. For instance, we found that the sensitivities of this model's elements with respect to the Y-parameters were fairly benign (thus, the villain in the piece was the ill conditioning of the Y-parameters with respect to the S-parameters). If Y-parameters could be measured directly, one could model nonlinear transistors much more accurately.

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New Tunable Phase Shifters Using Perturbed Dielectric Image Lines

Ming-yi Li and Kai Chang

***Abstract*—This paper presents new tunable phase shifters using perturbed dielectric image lines (DIL's). The propagation constant in the DIL was perturbed by a movable metal reflector plate installed in parallel with the ground plane of the DIL. The phase shift was thus controlled and adjusted by varying the perturbation spacing between the DIL and movable reflector plate at a given operating frequency. A rigorous hybrid-mode analysis was used for calculating the dispersion of propagation constants in the perturbed DIL, and then for designing tunable phase shifters. *Ka*-band tunable phase shifters have been designed, fabricated, and tested. Measurement results agree well with theoretical predictions. The device is especially useful for millimeter-wave applications where traditional phase shifters are lossy.**

***Index Terms*—Dielectric image lines, millimeter waves, tunable phase shifters.**

I. INTRODUCTION

Dielectric image lines (DIL's) have reduced losses compared to microstrip lines at millimeter-wave frequencies since most of the signal travels in the low-loss dielectric region [1]. This structure was recently proposed for feeding the aperture-coupled microstrip-patch antenna arrays [2]–[4], and overcomes the high conduction loss problem of microstrip lines at millimeter-wave frequencies. A phase shifter is one of the important control circuits used extensively at microwave and millimeter-wave frequencies. Traditional phase shifters use solid-state or ferrite devices. In this paper, new tunable phase shifters using DIL's are described. The DIL can be transformed to rectangular waveguide or microstrip line using transitions.

In a DIL, the electromagnetic (EM) signal travels mainly inside the dielectric and can be perturbed in several ways. The changing of propagation constants of an EM field in the DIL can be applied

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The authors are with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128 USA.
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